The Elasticity of Trade:
Estimates and Evidence

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ABSTRACT ——————————————————————————————

Quantitative results from a large class of structural gravity models of international trade depend critically on a single parameter governing the elasticity of trade with respect to trade frictions. We provide a new method to estimate this elasticity and illustrate the merits of our approach relative to the estimation strategy of Eaton and Kortum (2002). We employ this method on new disaggregate price and trade flow data for 123 developed and developing countries. Our benchmark estimate for all countries is approximately 4.22, nearly 50 percent lower than the alternative estimation strategy would suggest. This difference implies a doubling of the measured welfare costs of autarky across a large class of widely used trade models.

1. Introduction

Quantitative results from a large class of models of international trade depend critically on a single parameter that governs the elasticity of trade with respect to trade frictions.[1] To illustrate how important this parameter is, consider three examples: Anderson and van Wincoop (2003) find that the estimate of the tariff equivalent of the U.S.-Canada border varies between 48 and 19 percent depending upon the assumed elasticity of trade with respect to trade frictions. Yi (2003) points out that observed reductions in tariffs can explain almost all or none of the growth in world trade depending upon this elasticity. Arkolakis, Costinot, and Rodrguez-Clare (2009) argue that this parameter is one of only two parameters needed to measure the welfare cost of autarky in a large and important class of trade models. Therefore, this elasticity is key to understanding the size of the frictions to trade, the response of trade to changes in tariffs, and the welfare gains or losses from trade.

Estimating this parameter is difficult because quantitative trade models can rationalize small trade flows with either large trade frictions and small elasticities, or small trade frictions and large elasticities. Thus, one needs satisfactory measures of trade frictions *independent* of trade flows to estimate this elasticity. Eaton and Kortum (2002) provided an innovative and simple solution to this problem by arguing that, with product-level price data, one could use the maximum price difference across goods between countries as a proxy for bilateral trade frictions. The maximum price difference between two countries is meaningful because it is bounded by the trade friction between the two countries via simple no-arbitrage arguments.

We build on the approach of Eaton and Kortum (2002) and develop a new method to estimate this elasticity under the same data requirements. The argument for a new method above and beyond that of Eaton and Kortum (2002) is that their approach results in estimates that are biased upward by economically significant magnitudes. We illustrate this by performing a simple monte carlo experiment by discretizing the Eaton and Kortum (2002) model, simulating trade flows and product-level prices under an assumed elasticity of trade, and then applying their approach on artificial data. We find that one cannot recover the true elasticity of trade and that the estimates are biased upward by economically significant magnitudes.

The main reason why the approach of Eaton and Kortum (2002) fails to recover the true parameter is because the sample size of prices (typically 50-70 depending on the data set) is small relative to the number of goods in the economy. This is a problem because the

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[1]These models include Krugman (1980), Anderson and van Wincoop (2003), Eaton and Kortum (2002), and Melitz (2003) as articulated in Chaney (2008), which all generate log-linear relationships between bilateral trade flows and trade frictions.

probability that the max operator over a small sample of prices actually recovers the true trade cost is close to zero and the estimated trade cost will always be less than the true trade cost. Because the trade costs are almost always underestimated, this leads to systematic upward estimates of the elasticity of trade.

We develop a new method to estimate this elasticity when the sample size of prices is small. Our approach exploits the ability to use observed bilateral trade flows to recover all sufficient parameters to simulate trade flows and prices as a function of the parameter of interest. Given our ability to simulate these objects, we employ a simulated method of moments estimator that minimizes the distance between the parameter from the approach of Eaton and Kortum (2002) on real and artificial data. We explore the properties of this estimator numerically using simulated data and show that it can recover the true elasticity of trade in contrast to the alternative. More importantly, we formally argue that the estimator is proportional to the parameter of interest, with the proportionality factor converging to unity as the sample size of prices tends to infinity.

We apply our method to a new and unique data set. The new data set we employ contains disaggregate price and trade flow observations for 123 countries representing 98 percent of world GDP. The innovative feature of this data set is its coverage of developing countries. Previous estimates of this elasticity often come from small samples of developed countries.[2] Thus, the applicability of these estimates in the analysis of trade with both developed and developing countries is an important issue we can address.

Although we employ retail price data in our estimation procedure, we show that the resulting elasticity of trade estimates are not tainted by the presence of country-specific sales taxes, mark-ups and distribution costs. We present variants of the Ricardian model of trade that feature these market frictions, and we show that the frameworks yield identical estimating equations for the elasticity of trade as our benchmark. The simple intuition behind this result is that, should relative retail prices reflect various mark-ups in a multiplicative fashion, these mark-ups are also reflected in the estimates of trade costs, which employ the price data, and thus they perfectly cancel out in all estimating equations.

We also tackle measurement error and aggregation bias in price data, which arguably affect estimates of the elasticity of trade. Given our simulation approach, which makes use of a structural model of international trade, we are able to address measurement error by simulating prices of varieties with log-normal distributed errors. Aggregation bias, in turn, is an issue, because prices for our large sample of countries are reported at a so-called "basic-

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[2]See, for example, Head and Ries (2001) for the United States and Canada, Baier and Bergstrand (2001) and Eaton and Kortum (2002) for OECD countries, or the survey of these and several other studies in Anderson and van Wincoop (2004).

heading level". The price of a basic heading reflects an average price across a set of varieties of a particular good, thus potentially washing away extreme price differences across countries, which are necessary to obtain estimates of trade barriers. Consequently, we conduct robustness analysis by applying our approach to a more detailed cross-country product-level price database, provided by the European Intelligence Unit.

The benchmark estimate arising from our proposed simulated method of moments approach using new ICP price and trade flows data for 2004 is approximately 4.22. In contrast, the approach of Eaton and Kortum (2002) would yield estimates between 7.5 and 9.5, depending on whether the max or the second order statistic is used to approximate the trade friction.[3] We also apply our method to the data set of Eaton and Kortum (2002), which features only developed countries, and estimate the elasticity of trade to be approximately 3.93. This is in contrast to their preferred estimate of 8.28. Thus, our results provide strong evidence that the estimated elasticity of trade is in the range of 4 not 7-9, as the approach of Eaton and Kortum (2002) would suggest. Our results also provide suggestive evidence that this elasticity does not vary depending upon countries' level of development.

Why does this matter? As noted earlier, this matters because the measured welfare gains in quantitative models of trade depend critically on the elasticity of trade. Our new estimate of this elasticity implies a doubling of the percentage change in real income necessary to compensate a representative consumer for going to autarky, i.e. the welfare cost of autarky. Thus, while new heterogenous firm and production models may yield no larger welfare gains over simpler models as Arkolakis, Costinot, and Rodrguez-Clare (2009) argue, only with the structure of a heterogenous production model such as Eaton and Kortum (2002), could we have used both measurement and theory to arrive at a more robust and better estimate of the elasticity of trade and hence the welfare gains from trade.

2. Model

We analyze a version of the multi-country Ricardian model of trade introduced by Eaton and Kortum (2002). We consider a world with N countries, where each country has a tradable final goods sector. There is a continuum of tradable goods indexed by $j \in [0, 1]$.

Within each country i, there is a measure of consumers L_i. Each consumer has one unit of time supplied inelastically in the domestic labor market and enjoys the consumption of a

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[3]Our approach is robust to using either the max or the second order statistic, while the approach of Eaton and Kortum (2002) always generates larger estimates using the second order statistic.

CES bundle of final tradable goods with elasticity of substitution $\rho > 1$:

$$U_i = \left[\int_0^1 x_i(j)^{\frac{\rho-1}{\rho}} dj \right]^{\frac{\rho}{\rho-1}}$$

To produce quantity $x_i(j)$ in country i, a firm employs labor using a linear production function with productivity $z_i(j)$. Country i's productivity is in turn the realization of a random variable Z_i (drawn independently for each j) from its country-specific Fréchet probability distribution $F_i(z_i) = \exp(-T_i z_i^{-\theta})$. The country-specific parameter $T_i > 0$ governs the location of the distribution, thus higher values of it imply that a high productivity draw for any good j is more likely. The parameter $\theta > 1$ is assumed to be common across countries and if higher, it generates less variability within the distribution.[4]

Having drawn a particular productivity level, a perfectly competitive firm from country i incurs a marginal cost to produce good j of $w_i / z_i(j)$, where w_i is the wage rate in the economy. Shipping the good to a destination n further requires a per unit iceberg cost of $\tau_{ni} > 1$ for $n \neq i$, with $\tau_{ii} = 1$. We assume that cross-border arbitrage forces effective geographic barriers to obey the triangle inequality: For any three countries i, k, n, $\tau_{ni} \leq \tau_{nk}\tau_{ki}$. With these in mind, the marginal cost of production and delivery of good j from country i to destination n is given by:

$$p_{ni}(j) = \frac{\tau_{ni} w_i}{z_i(j)}.$$

International markets are perfectly competitive, so consumers in destination n would pay $p_{ni}(j)$, should they decide to buy good j from country i. Thus, the actual price consumers in n pay for good j is the minimum price across all sources k:

$$p_n(j) = \min_{k=1,\ldots,N} \left\{ p_{nk}(j) \right\}.$$

Substituting the pricing rule into the productivity distribution allows us to obtain the following price index for each destination n:

$$P_n = \gamma \left[\sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta} \right]^{-\frac{1}{\theta}}. \tag{1}$$

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[4]In our quantitative analysis, we estimate values for this parameter for different sets of countries and conclude that they are fairly similar, a finding that supports this assumption.

In the above equation,

$$\gamma = \left[\Gamma \left(\frac{\theta + 1 - \rho}{\theta} \right) \right]^{\frac{1}{1-\rho}},$$

where Γ is the Gamma function and parameters are restricted such that $\theta > \rho - 1$.

Furthermore, let X_n be country n's expenditure on final goods, of which X_{ni} is spent on goods from country i. Since there is a continuum of goods, computing the fraction of income spent on imports from i, X_{ni}/X_n, can be shown to be equivalent to finding the probability that country i is the low-cost supplier to country n given the joint distribution of efficiency levels, prices, and trade costs for any good j. The expression for the share of expenditures that country n spends on goods from country i or, as we will call it, the trade share is

$$\frac{X_{ni}}{X_n} = \frac{T_i(\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^N T_k(\tau_{nk} w_k)^{-\theta}}. \tag{2}$$

Note that the sum across k for a fixed n must add up to one.

Expressions (1) and (2) allow us to relate observed expenditure shares to bilateral trade frictions and the price indices of each trading partner via the following equation:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{\tau_{ni} P_i}{P_n} \right)^{-\theta}. \tag{3}$$

Expression (3) is not particular to the model of Eaton and Kortum (2002). Several popular models of international trade relate trade shares, prices and trade costs in the same exact manner. These models include the Armington framework of Anderson and van Wincoop (2003), as well as the monopolistic competition frameworks of firm heterogeneity by Melitz (2003) and Chaney (2008), and their homogeneous foundation introduced by Krugman (1980). Appendix 11 describes the models in detail and derives expression (3) for each one of them.

2.1. The Elasticity of Trade

Consider expression (3). The parameter of interest is θ. To see the parameter's importance, take logs of equation (3) yielding

$$\log \left(\frac{X_{ni}/X_n}{X_{ii}/X_i} \right) = -\theta \log(\tau_{ni}) - \theta \log(P_i) + \theta \log(P_n).$$

As this expression makes clear, θ controls how a change in the bilateral trade costs, τ_{ni}, will change bilateral trade between two countries. This elasticity is important because if one wants to understand how a bilateral trade agrement will impact aggregate trade or simply understand the magnitude of the trade friction between two countries, then a stand on this elasticity is necessary. This is what we mean by the elasticity of trade.

This elasticity takes on an even larger role than merely controlling trade's response to trade frictions. Arkolakis, Costinot, and Rodrguez-Clare (2009) argue further that this elasticity is one of only two objects that control the welfare gains from trade in the same class of models we discussed above. Thus this elasticity is absolutely critical in any quantitative study of international trade in a large class of models.

3. Estimating θ: Eaton and Kortum's (2002) Approach

Equation (3) suggests that one could easily identify θ if one had data on trade shares, aggregate prices, and trade costs. However, the identification problem that one faces is that trade costs are not observed.[5] That is one can rationalize small trade flows with either large trade frictions and small elasticities or small trade frictions and large elasticities. Thus, one needs satisfactory measures of trade frictions *independent* of trade flows to estimate this elasticity. Eaton and Kortum (2002) employ an innovative approach to approximate trade costs τ_{ni}. They exploit *disaggregate* price information across countries by arguing that the maximum price difference between two countries bounds the trade costs between the two countries via simple no-arbitrage arguments.

To illustrate Eaton and Kortum's (2002) argument, consider the following example: Suppose there are two countries (home and foreign) and two goods (TVs and DVD players) and prices for each of these goods are observed as in Table 3.

Table 3: Two countries and Two Prices

	TV's	DVD
Price Home	150	125
Price Foreign	100	100

Table 3 provides the following information about trade costs between the two countries.

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[5]It should be noted that price indices as such are also not observed. However, given disaggregate price data, one can approximate exact CES price indices through geometric averages of prices of individual products. In later sections, we show that this approximation does not bias the estimates for θ when the sample size of prices is large.

First, notice that if the trade cost $\tau_{h,f} < 1.50$, then someone in the home country could simply import TV's from the foreign country and sell them at a profit and bid away the price difference. Thus the trade friction is no less than 1.50. Notice—and this is a key point to understand for our argument—that this is only a lower bound. Only if the home country actually imports TV's does one know that the trade friction is 1.50. If the home country is not importing TV's then the trade friction may be greater than or equal to 1.5.

In general, it must be the case that for a given good j, $\frac{p_n(j)}{p_i(j)} \leq \tau_{ni}$; otherwise, there would be an arbitrage opportunity as described above. This suggests that an estimate of τ_{ni} is the maximum of relative prices over goods j. To summarize, Eaton and Kortum's (2002) proxy for τ_{ni}, in logs, is

$$\log \hat{\tau}_{ni} = \max_j \left\{ \log \left(p_n(j) \right) - \log \left(p_i(j) \right) \right\}, \tag{4}$$

where the max operator is over all j goods.

Using (4), trade data, and the geometric average over disaggregate price data to approximate P_i, Eaton and Kortum (2002) exploit the structural relationship in (3) to estimate θ. Details specific to their estimate are that they use a method of moments estimator and the second order statistic rather than the max. This approach yields their preferred estimate of 8.28. Table 5 summarizes estimates of θ and the standard errors associated with each approach.

Table 5: Summary of Eaton and Kortum (2002) Results, Second Order Statistic

Statistic	Method of Moments	Least Squares	Least Squares
Intercept	—	—	-2.18 (0.40)
Slope	-8.28 (0.18)	-8.03 (0.18)	-4.55 (0.66)
SSE	1403	1395	1286
TSS	1463	1463	1463
# Obsv.	342	342	342

4. Monte Carlo Evidence

In this section, we study Eaton and Kortum's (2002) approach to estimating θ as described in section 3. We study their approach by simulating a data set under an assumed value for θ and see if Eaton and Kortum's (2002) approach can recover the true value of θ that generated the data. Our main finding is that their approach cannot and that their estimates

of θ are biased upward by quantitatively significant amounts. We argue that this failure arises because of a limited sample of prices to estimate trade costs.

4.1. Simulation Approach

We want to simulate a data set from a stochastic Ricardian model along the lines of Eaton and Kortum (2002) that resembles data.[6] We use the approach described in the steps below. This simulation approach also provides the foundations for the simulated method of moments estimator we propose in the next section.

Step 1.—We estimate parameters for the country specific Fréchet distributions and trade costs from bilateral trade flow data. We perform this step by following Eaton and Kortum (2002) and Waugh (2009) and deriving the following gravity equation from equation (2) by dividing the bilateral trade share by the importing country's home trade share,

$$\log\left(\frac{X_{ni}/X_n}{X_{nn}/X_n}\right) = S_i - S_n - \theta \log \tau_{ni}, \tag{5}$$

in which S_i is defined as $\log\left[w_i^{-\theta}T_i\right]$. Note that this is a different equation than that used to estimate θ in (3) which is derived by dividing the bilateral trade share by the exporting country's home trade share. S_is are recovered as the coefficients on country-specific dummy variables given the imposed restrictions on how trade costs can covary across countries. Following the arguments of Waugh (2009), trade costs take the following functional form:

$$\log(\tau_{ni}) = d_k + b_{ni} + ex_i + \nu_{ni}. \tag{6}$$

Here, trade costs are a logarithmic function of distance, where d_k with $k = 1, 2, ..., 6$ is the effect of distance between country i and n lying in the kth distance intervals.[7] b_{ni} is the effect of a shared border in which $b_{ni} = 1$, if country i and n share a border and zero otherwise. The term ex_i is an exporter fixed effect and allows for the trade cost to vary in level depending upon the exporter. We assume ν_{ni} reflects barriers to trade arising from all other factors and is orthogonal to the regressors. We use least squares to estimate equations (5) and (6) to the bilateral trade shares.

Before proceeding, note that what we are doing here is exploiting the fact that we can estimate all necessary parameters to simulate trade flows and prices up to a constant, θ. This

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[6] In all the monte-carlo experiments, we use the trade data in Eaton and Kortum (2002) in Step 1 of the simulation procedure. Section 7.2.1 describes their data in more detail.

[7] Intervals are in miles: $[0, 375)$; $[375, 750)$; $[750, 1500)$; $[1500, 3000)$; $[3000, 6000)$; and $[6000, \text{maximum}]$. Our results are robust to alternative trade cost specifications such as the one in Eaton and Kortum (2002).

allows us to be able to simulate data as a function of the parameter θ only. The relationship is obvious in the estimation of trade barriers since τ_{ni} is scaled by θ in (5). To see that we can simulate prices as a function of θ only, notice that for any good j, $p_{ni}(j) = \tau_{ni} w_i / z_i(j)$. Thus, rather than simulating productivities, it is sufficient to simulate the inverse of marginal costs of production $u_i(j) = z_i(j)/w_i$. Since productivities are distributed according to the Fréchet distribution $F_i(z_i) = \exp(-T_i z_i^{-\theta})$, it is easy to verify that u_i is distributed according to $G_i(u_i) = \exp(-T_i w_i^{-\theta} u_i^{-\theta})$.[8] From the gravity equation in (5), notice that $S_i = \log(T_i w_i^{-\theta})$. Thus, having obtained the coefficients S_i, we can simulate the inverse of marginal costs $u_i(j)$ using $G_i(u_i) = \exp(-\tilde{S}_i u_i^{-\theta})$, where $\tilde{S}_i = \exp(S_i)$, and easily obtain price observations $p_{ni}(j) = \tau_{ni} u_i(j)^{-1}$. Finally, we can easily simulate trade shares according to expression (2) once again using estimated coefficients S_i and bilateral trade barriers τ_{ni}, having specified a value for the crucial elasticity parameter θ.

Step 2.—With an assumed θ, the estimated $\hat{S}_i$ parameterize the Fréchet distributions for each country and the level of trade costs. In the simulations that follow, we set θ equal to 8.28— the preferred estimate of Eaton and Kortum (2002). With the parameterized distributions and trade costs, we can then simulate the model.

To simulate the model, we assumed there is a large number (100,000) of potentially tradable goods. For each country, good-level efficiencies are drawn from the country-specific distribution and assigned to the production technology for each good. Then, for each importing country and each good, the low-cost supplier across countries is found, realized prices are recorded, and the aggregate bilateral trade shares are computed.

Step 3.—From the realized prices, a subset of goods common to all countries is defined and the subsample of prices are recorded, i.e. we are acting as if we were collecting prices for the international organization that collects the data. We added disturbances to the predicted trade shares with the disturbances drawn from a mean zero normal distribution with the standard deviation set equal to the standard deviation of the residuals, ν_{ni}, from Step 1.

Step 4.—Given the prices and trade shares, we then employ the estimation strategy suggested by Eaton and Kortum (2002).

We should note that the most important variable in the simulation is the sample size of the prices. It is important because small samples of prices will lead to significantly biased estimates of θ. In our baseline simulation, we use a sample size of 50. This is the same sample size of prices used in Eaton and Kortum (2002).

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[8]See Appendix 13.1 for formal proof.

4.2. Monte Carlo Results

Table 3 presents the results from the steps outlined above. The columns of Table 3 present the mean and median estimates of θ over the 100 simulations. The rows present different estimation approaches, i.e. simple least squares and method of moments (the preferred approach of Eaton and Kortum (2002)) all with intercepts suppressed. The top panel uses the first order statistic. The bottom panel uses the second order statistic as used in the preferred approach of Eaton and Kortum (2002).

Table 3: Monte Carlo Results, True θ = 8.28

Approach	Mean Estimate of θ (S.D)	Median Estimate of θ
First Order Statistic		
Least Squares	12.1 (0.60)	12.1
Method of Moments	12.5 (0.60)	12.5
Second Order Statistic		
Least Squares	14.7 (0.60)	14.7
Method of Moments	15.2 (0.60)	15.2

Note: S.D. is the standard deviation . In each simulation there are 19 countries and 100,000 goods. Only 50 realized prices are randomly sampled and used to estimate θ. 100 simulations performed.

The key result from Table 3 is that the estimates of θ are significantly larger than the true θ that generated the data. As discussed, the underlying θ was set equal to 8.28 and the estimated θ's in the simulation are between 12 and 15. This suggests the approach of Eaton and Kortum (2002) cannot recover the assumed value of θ and that this approach generates estimates that are biased upward by quantitatively significant amounts.

5. Why the Failure?

The main reason why the estimator proposed by Eaton and Kortum (2002) cannot uncover the true value of θ is that the sample of prices is not large enough. Below we analytically show the bias present in their estimator and show that only as the sample size of prices becomes very large does their estimator converge to the true value.

To illustrate these properties, we will compute the expected value of their estimate. First, consider the estimating equation used by Eaton and Kortum (2002) where the parameter of

interest is β:

$$\log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right) = -\beta\left(\log\hat{\tau}_{ni} + \log\hat{P}_i - \log\hat{P}_n\right),\tag{7}$$

$$\text{where}\quad \log\hat{\tau}_{ni} = \max_{z\in Z}\left\{\log p_n(z) - \log p_i(z)\right\},$$

$$\text{and}\quad \log\hat{P}_i = \frac{1}{Z}\sum_{z=1}^{Z}\log(p_i(z)),$$

where Z is the total number of goods whose prices are observed in countries n and i.[9] The first part of the expression approximates the barrier i incurs to sell to n, while the second approximates the difference in the exact price indices in the two countries. A method of moments estimator for β gives the following estimate of the parameter, using all bilateral country pairs in the sample:

$$\beta = -\frac{\sum_n\sum_i\log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n\sum_i\left(\log\hat{\tau}_{ni} + \log\hat{P}_i - \log\hat{P}_n\right)}\tag{8}$$

Treating the prices as random draws from their given distributions, we then compute the expected value of β.

Definition 1 *Define the following objects*

- *Let $\epsilon_{ni} = \theta[\log p_n - \log p_i]$ be the log price difference of a good between country n and country i, multiplied by θ.*

- *Let $f_T(\epsilon_{ni})$ be the truncated probability distribution of $\epsilon_{ni}\in[-\theta\log(\tau_{in}),\theta\log(\tau_{ni})]$.*

- *Let $F_{\max}(\epsilon_{ni};Z) = F_T(\epsilon_{ni})^Z$ be the cdf of $\max(\epsilon_{ni})$, given a sample $Z\geq 1$ of prices.*

- *Let $f_{\max}(\epsilon_{ni};Z) = ZF_T(\epsilon_{ni})^{Z-1}f_T(\epsilon_{ni})$ be the pdf of $\max(\epsilon_{ni})$ given a sample $Z\geq 1$ of prices.*

Given these definitions Proposition 1 characterizes the expected value of β.

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[9]In all arguments that follow, we index goods by the efficiency associated with producing them, z.

Proposition 1 *Consider an economy of N countries with a sample of Z prices observed. The expected value of β is*

$$E(\beta) = -\theta \frac{\sum_n \sum_i \log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n \sum_i (\Psi_{ni}(Z) - \Omega_{ni})}, \tag{9}$$

with

$$1 \leq -\frac{\sum_n \sum_i \log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n \sum_i (\Psi_{ni}(Z) - \Omega_{ni})} \tag{10}$$

where:

$$\Omega_{ni}(\boldsymbol{S}, \theta \log(\tau_{ni}), \theta \log(\tau_{in})) \equiv \int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f_T(\epsilon_{ni}) d\epsilon_{ni},$$

$$\Psi_{ni}(Z; \boldsymbol{S}, \theta \log(\tau_{ni}), \theta \log(\tau_{in})) \equiv \int_{-\theta \log(\tau_{in})}^{\theta \log(\tau_{ni})} \epsilon_{ni} f_{\max}(\epsilon_{ni}; Z) d\epsilon_{ni},$$

The proof of Proposition 1 can be found in appendix 13.2. Intuitively, the proposition establishes the fact that the estimate of β exceeds the true value of the elasticity of trade θ. To see this, first consider expression (9). We refer to the numerator, country i's share in country n relative to i's share at home, as country i's normalized import share in country n. The assumed triangle inequality in trade barriers ensures that this staistic never exceeds unity. Hence, the numerator is positive when multiplied by -1. For any $Z \geq 1$, the denominator is non-negative. This comes from the fact that the expectation of the maximum of $Z \geq 1$ realizations of a random variable can never be below the sample mean of the realizations.

In order to understand why the denominator can never exceed the numerator (inequality in expression (10)), consider a sample of prices, Z, and notice that the relative trade shares in the numerator are not affected by Z. With respect to the denominator, Ω_{ni} is simply the sample (geometric) mean of prices, which is an unbiased and consistent estimator of the true mean of prices. More importantly, the denominator contains Ψ_{ni}, which is the expectation of the maximum price difference between countries n and i. Due to the assumed triangle inequality in trade barriers, the true value of the maximum price difference is given by $\theta \log(\tau_{ni})$. Its expectation, Ψ_{ni}, depends crucially on the sample size of prices $Z \geq 1$ from which the maximum price difference is computed. Suppose this sample size Z is finite. Then, the probability that the maximum price difference lies below some value $\tilde{\epsilon}_{ni} < \theta \log(\tau_{ni})$ is $F_{\max}(\tilde{\epsilon}_{ni}, Z) = F_T(\tilde{\epsilon}_{ni})^Z$. This probability is strictly less than unity, unless

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all the mass of the distribution of price differences is concentrated at $\theta \log(\tau_{ni})$. This means that, for a finite sample of prices, Ψ_{ni} always lies below the true value of the maximum price difference, yielding a strict inequality in expression (10). Hence, the maximum price difference is always underestimated, yielding an estimate of β that strictly exceeds the value of the elasticity of trade, θ.

To concretely illustrate this, reconsider the same example from Section 3 but with three goods (TVs and DVD players and Xbox's). Prices for TVs and DVD players are the same, but the price of the Xbox is 165 in the Home country and 100 in the foreign country. The new information suggests a new estimate of the trade cost to be 1.65. The previous estimate of $\tau_{h,f} = 1.50$ with only two prices is biased downward by 0.15 when three prices are considered.

A downward bias in the estimate of the true trade cost is essentially associated with a low $\Psi_{h,f}$. Since this object is in the denominator of (9), the proportionality factor is large. Hence β is well in excess of θ, namely the estimate of the elasticity of trade is biased upwards.

As the sample size, Z, becomes sufficiently large, however, the factor of proportionality converges to unity. Proposition 2, whose proof is in appendix 13.2, states the convergence result.

Proposition 2 *As $Z \to \infty$, $\beta \to \theta$, that is,*

$$plim_{Z \to \infty} - \frac{\sum_n \sum_i \log \left(\frac{X_{ni}/X_n}{X_{ii}/X_i} \right)}{\sum_n \sum_i (\Psi_{ni}(Z) - \Omega_{ni})} = 1.$$

Proposition 2 confirms that the methodology introduced by Eaton and Kortum (2002) can recover the true value of the elasticity of trade, only as the sample size of prices becomes extremely large.

To further advance this argument, we performed the same monte carlo exercise in the previous section but with 500, 5,000, and 50,000 sampled prices. Table 4 presents the results. Notice how the estimate of θ becomes less biased and begins to approach the true value of θ as the sample of prices becomes larger.

However, the rate of convergence is extremely slow; even with a sample size of $5,000$ the estimate of β is larger than the value generating the data. Only when 50,000 prices are sampled—one half of all goods in the economy—does the estimate converge to the true value. This suggests that data requirements needed to yield an unbiased estimate of θ are extreme.

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Sample Size of Prices	Mean Estimate of β (S.D.)	Median Estimate of θ
50	12.14 (0.60)	12.15
500	9.41 (0.22)	9.40
5,000	8.47 (0.08)	8.47
50,000	8.29 (0.06)	8.29

Note: S.D. is the standard deviation. In each simulation there are 19 countries and 100,000 goods. The results reported use least squares with the constant suppressed. 100 simulations performed.

6. Solution: A New Approach To Estimating θ

In this section we suggest a new approach to estimating θ and discuss its performance on simulated data.

6.1. The Idea

Our idea is to exploit (a) the ability to simulate from the model as a function of θ only and (b) Proposition 1. Point (a) allows us to fix a θ and then compute any moments we desire from the model. Point (b) tells us which moments are informative as it shows that that the estimate β is *proportional* to the parameter of interest θ. So our estimation matches the data moment β to the β implied by the model simulated under a known θ. Then because of the monotonicity implied by Proposition 1, the known θ must be the unique value that satisfies the moment condition specified.

Figure 1 illustrates this idea. On the x-axis are the values β_k with k denoting the order statistic used (see below). The y-axis describes the values of θ. The two upward sloping lines illustrate how the value β_k varies linearly with θ as we proved in Proposition 1.

Our estimation approach basically uses simulation of the model to quantitatively characterize the mapping between θ and β_k, i.e. the lines in figure 1. Then given the observed moments seen, this mapping tell's us about the θ that is consistent with the observed moments.
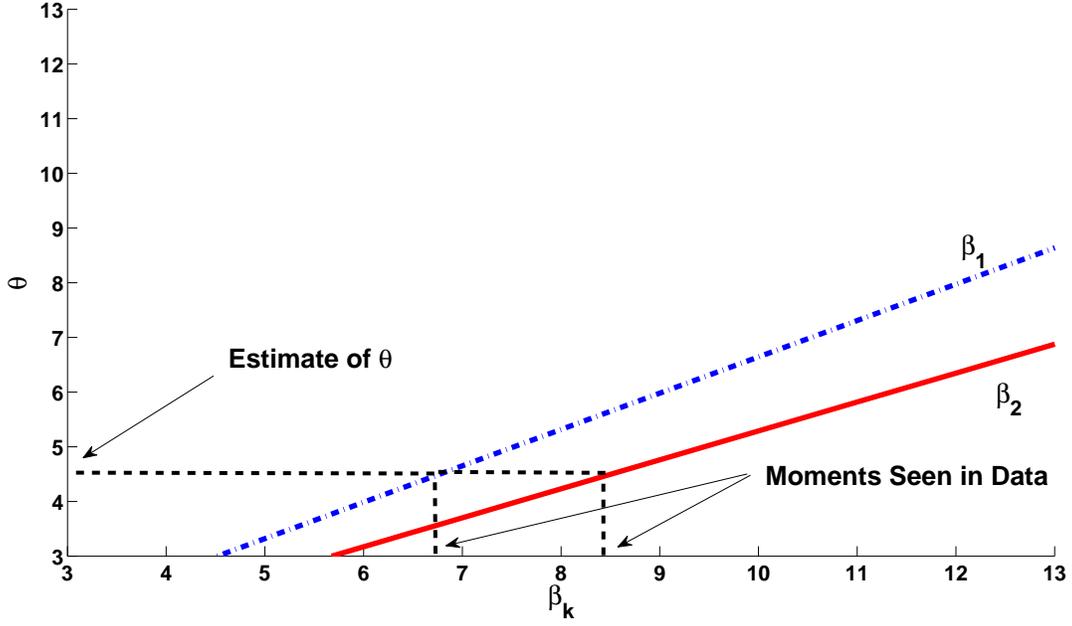
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Figure 1: Schematic of Estimation Approach

6.2. Simulation

Steps 1-3 in section 4.1 outline our approach to simulate data, such as trade shares and good-level prices, as a function of our parameter of interest θ.

6.3. Moments

Here we will define the moments of interest. Define β_k as method of moment estimator using the kth order statistic over good-level price data. Then the moments we are interested in are:

$$\beta_k = -\frac{\sum_n \sum_i \log\left(\frac{X_{ni}/X_n}{X_{ii}/X_i}\right)}{\sum_n \sum_i \left(\log \hat{\tau}_{ni}^k + \log \hat{P}_i - \log \hat{P}_n\right)}, \quad k = 1, 2 \tag{11}$$

Where $\hat{\tau}_{ni}^k$ is computed as the kth order statistic over good-level price data between country n and i. The moments we are going to use are the Eaton and Kortum (2002) method of moments estimate use both the first and second order statistic. We also consider an exactly identified estimation using β_1 as the only moment.

We will denote the simulated moments as $\beta_1(\theta, u_s)$ and $\beta_2(\theta, u_s)$ which come from the analogous regression as in (11), except that the trade shares, estimated trade costs, and estimated

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price indices are from simulated data as a function of θ and depend upon a vector of random variables u_s associated with a particular simulation s. There are three components to this vector. First, there are the random productivity draws for production technologies for each good and each country. The second component is the set of goods that are sampled from all countries. The third component mimics the residuals ν_{ni} from equation (5) and described in Section 4.1.

Stacking our data moments and averaged simulation moments gives us the following zero function:

$$y(\theta) = \begin{bmatrix} \beta_1 - \frac{1}{S} \sum_{s=1}^{S} \beta_1(\theta, u_s) \\ \\ \beta_2 - \frac{1}{S} \sum_{s=1}^{S} \beta_2(\theta, u_s) \end{bmatrix}. \tag{12}$$

6.4. Estimation Procedure

We base our estimation procedure on the moment condition:

$$E\left[y(\theta_o)\right] = 0,$$

where θ_o is the true value of θ. Thus our simulated method of moments estimator is

$$\hat{\theta} = \arg \min_{\theta} \left[y(\theta)' \, \mathbf{W} \, y(\theta)\right], \tag{13}$$

where $\mathbf{W}$ is a 2×2 weighting matrix which we discuss below. The idea behind this moment condition is that though α and β will be biased away from 0 and θ, the moments $\alpha(\theta, u_s)$ and $\beta(\theta, u_s)$ will be biased by the same amount when evaluated at θ_o, in expectation. Viewed in this language, our moment condition is closely related to the estimation of bias functions discussed in MacKinnon and Smith (1998) and is closely related to indirect inference as discussed in Smith (2008).[10]

For the weighting matrix, we use the inverse of the estimated variance-covariance matrix Ω of the moments α and β estimated from the data.[11] To compute Ω, we used a simple bootstrap procedure outlined in the following steps.

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[10] A key issue in MacKinnon and Smith (1998) is how the bias function behaves. Proposition 1 show that the bias function is independent of the parameter of interest and thus is well behaved.

[11] This weighting matrix makes sense for the following arguments: First, the optimal weighting matrix should be the inverse of the variance-covariance matrix of $y(\theta_o)$. Second, note that $\text{Var}(y(\theta_o)) = \text{Var}([\alpha, \beta]) + \frac{1}{S}\text{Var}([\alpha(\theta_o, u_s), \beta(\theta_o, u_s)]) = (1 + \frac{1}{S})\text{Var}([\alpha, \beta])$. Thus the appropriate weighting matrix is $\{(1 + \frac{1}{S})\text{Var}([\alpha, \beta])\}^{-1}$. See Davidson and MacKinnon (2004) for more details.

Step 1.— We computed residuals v_{ni} implied by the estimator in (11) and the fitted values, we resampled the residuals v_{ni} with replacement and generated a new set of data using the fitted values. Using the data constructed from each resampling b, we computed new estimates β_{1b} and β_{2b}.

Step 2.—Define the difference between the bootstrap generated moments and data moments as:

$$m^b = \begin{bmatrix} \beta_1 - \beta_{1b} \\ \beta_2 - \beta_{2b} \end{bmatrix} \tag{14}$$

we then computed the variance-covariance matrix as

$$\Omega = \frac{1}{B} \sum_{b=1}^{B} (m^b) \times (m^b)' \tag{15}$$

then the weighting matrix W is set equal to Ω^{-1}.

We compute standard errors using a bootstrap technique. Here it is important to take into account both sampling error and simulation error. To account for sampling error, each bootstrap b replaces the moments α and β with bootstrap generated moments β_1^b and β_2^b. Then to account for simulation error, a new seed is generating a new set of model generated moments: $\frac{1}{S} \sum_{s=1}^{S} \beta_1(\theta, u_s)^b$ and $\frac{1}{S} \sum_{s=1}^{S} \beta_2(\theta, u_s)b$. Then defining $y^b(\theta)$ as the difference in moments for each b as in (14), we solve for

$$\hat{\theta}^b = \arg \min_{\theta} \left[y^b(\theta)' \, \mathbf{W} \, y^b(\theta) \right]. \tag{16}$$

We repeat this exercise 100 times and compute the estimated standard error of our estimate of $\hat{\theta}$ as

$$\text{S.E.}(\hat{\theta}) = \left[\frac{1}{100} \sum_{b=1}^{100} (\hat{\theta}^b - \hat{\theta})(\hat{\theta}^b - \hat{\theta})' \right]^{\frac{1}{2}} \tag{17}$$

This procedure to constructing standard errors is similar in spirit to the approach employed in Eaton, Kortum, and Kramarz (2008) who use a simulated method of moments estimator to estimate the parameters of a similar trade model from the performance of French exporters.

6.5. Performance on Simulated Data

In this section, we evaluate the performance of our estimation approach using simulated data when we know the true value of θ. In all the results that followed, we set the true value of θ equal to 8.28.

Table 5 presents the results from the following exercise. We generated a artificial data set with true value of θ equal to 8.28 and then applied our estimation routine.

The first row presents our simulated method of moments estimate which is 8.22 with a standard error of 0.34. This is effectively the true value of θ generating the data.

Table 5: Estimation Results With Simulated Data

Estimation Approach	Mean Estimate of θ	Standard Deviation
Overidentified	**True θ = 8.28**	
SMM	8.22	0.34
Method of Moments, β_1	12.37	0.61
Method of Moments, β_2	15.09	0.59
Exactly Identified	**True θ = 8.28**	
SMM	8.19	0.41
Method of Moments, β_1	12.37	0.61

Note: In each simulation there are 19 countries and 100,000 goods and 100 simulations performed. The value k refers to the order statistic employed. The over-identified case uses $k = 1$ and $k = 2$. The exactly-identified case uses $k = 1$.

To emphasize the performance of our estimator, the next two rows of Table 5 present the approach of Eaton and Kortum (2002) (and which also correspond with the moments used). Though not surprising given the discussion above, this approach generates estimates of θ that are significantly (in its economic meaning) higher than the true value of θ of 8.28.

The final two rows present the exactly identified case when we use only one moment to estimate θ. In this case we used β_1. Similar to the over-identified case, our simulated method of moments estimate which is 8.19 with a standard error of 0.41. Again, this is effectively the true value of θ generating the data.

We view these results as evidence supporting our estimation approach and empirical estimate of θ presented in Section 7.

7. Empirical Results

In this section, we apply our estimation strategy described in section 6 to several different data sets. The key finding of this section is that our estimation approach yields an estimate around 4.5 in contrast to previous estimation strategies which yield estimates around 8.

7.1. Baseline Results Using New ICP 2005 Data

7.1.1. New ICP 2005 Data

Our sample contains 123 countries. We use trade flows and production data for the year 2004 to construct trade shares. The price data used to compute aggregate price indices and proxies for trade costs comes from basic heading level data from the 2005 round of the International Comparison Programme (ICP). The ICP collects price data on goods with identical characteristics across retail locations in the participating countries during the 2003-2005 period.[12] The basic heading level represents a narrowly-defined group of goods for which expenditure data are available. In the data set there are a total of 129 basic headings and we reduce it to 62 based on its correspondence with the trade data employed. Appendix 12 provides more details.

On its own this data set provides two contributions to the existing analysis. First, because this is the latest round of the ICP the measurement issues are probably less severe than previous rounds. Furthermore, this data set includes both developed and developing countries and allows us to study questions regarding how the elasticity of trade may vary depending upon countries' income levels.

7.1.2. Results—New ICP 2005 Data

Table 6 presents the results.

The top panel reports results for the overidentified estimation and the underlying moments used (Eaton and Kortum (2002) estimator using both the first and second order statistic). The bottom panel reports the results for the exactly identified estimation and the underlying moment used (Eaton and Kortum (2002) using the first order statistic. In both instances, our estimation procedure delivers estimates of around 4.22 with a fairly small standard error. This is in contrast to estimates using the Eaton and Kortum (2002) methodology, which vary between 7.5 to 9.5 depending upon if the first order statistic or second order statistic is used.

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[12]The ICP Methodological Handbook is available at http://go.worldbank.org/MW520NNFK0.

Table 6: Estimation Results With 2005 ICP Data

Estimation Approach	Estimate of θ	Standard Error
Overidentified		
SMM	4.19	???
Method of Moments, β_1	7.75	0.03
Method of Moments, β_2	9.61	0.03
Exactly Identified		
SMM	4.22	???
Method of Moments, β_1	7.75	0.03

7.2. Estimates Using Eaton and Kortum's (2002) Data

In this section, we apply our estimation strategy to the same data used in Eaton and Kortum (2002) as another check of our estimation procedure. Furthermore, because it includes only OECD countries it allows us to preliminarily consider if estimates from developed countries differ than estimates using data with developed and developing countries.

7.2.1. Eaton and Kortum's (2002) Data

Their data set consists of bilateral trade data for 19 OECD countries in 1990 and 50 prices of manufactured goods for all countries. The prices come from an earlier round of the ICP which considered only OECD countries. Similar to our data, the price data is at the basic heading level and is for goods with identical characteristics across retail locations in the participating countries.

7.2.2. Results—Eaton and Kortum's (2002) Data

Table 7 presents the results. The top panel reports results for the overidentified estimation and the underlying moments used (Eaton and Kortum (2002) estimator using both the first and second order statistic). The bottom panel reports the results for the exactly identified estimation and the underlying moment used (Eaton and Kortum (2002) using the first order statistic. In both cases, our estimation strategy generates results substantially below previous estimates; 3.9 in the exactly identified estimation relative to 6ish numbers when using the first order statistic. 4.5 relative to 8ish numbers when using the second order statistic. In

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all cases, the standard errors are fairly tight.

Estimation Approach	Estimate of θ	Standard Error
Overidentified		
SMM	4.47	0.10
Method of Moments, β_1	5.93	0.12
Method of Moments, β_2	8.28	0.11
Exactly Identified		
SMM	3.93	0.09
Method of Moments, β_1	5.93	0.12

7.3. Discussion

Our estimation results compare favorably with alternative estimates of θ which *do not* use the max over price data to approximate trade costs. For example, estimates of θ using firm level data as in Bernard, Eaton, Jensen, and Kortum (2003) and Eaton, Kortum, and Kramarz (2008) are in the range of 3.6 to 4.8—exactly in the range of values we find. Eaton and Kortum (2002) provide an alternative estimate of θ using wage data and find a value of 3.6. Burstein and Vogel (2009) estimate θ matching moments regarding the skill intensity of trade and find a value of 4. Simonovska (2010) uses a non-homothetic model of trade featuring variable mark-ups and calibrates θ to a level of 3.8 which allows her model to match average mark-ups in OECD countries.

Donaldson (2009) estimates θ as well and his approach is illuminating relative to the issues we have raised. His strategy to approximating trade costs is to study differences in the price of salt across locations in India. In principal, his approach is subject to our critique as well, i.e. how could price differences in one good be informative about trade frictions? However, he argues convincingly that in India salt was produced in only a few locations and exported everywhere. Thus by examining salt, Donaldson (2009) has found a "binding" good. Using this approach, he finds estimates in the range of 3.8-5.2, again consistent with the range of our estimates of θ.

Moreover, note that the estimates of θ when only OECD countries are considered (Eaton and Kortum's (2002) data) are similar to our baseline with a large number of developed and

developing countries. This evidence is suggestive that θ does not vary systematically across countries depending upon the level of development of the country.

Finally, it should be noted that the elasticity of trade, θ, is closely related to the elasticity of substitution between foreign and domestic goods, the Armington elasticity, which determines the behavior between trade flows and relative prices across a large class of models. Recently, Ruhl (2008) presents a comprehensive discussion of the puzzle regarding this elasticity. In particular, he argues that international real business cycle models need low elasticities, in the range of 1 to 2, to match the quarterly fluctuations in trade balances and the terms of trade, but static applied general equilibrium models need high elasticities, between 10 and 15, to account for the growth in trade following trade liberalization. Using very disaggregate data, Romalis (2007), Broda and Weinstein (2006), and Hummels (2001) provide estimates for the Armington elasticity parameter across a large number of industries. Romalis's (2007) estimates range between 4-13, Hummels's (2001) estimates range between 3-8, while the most comprehensive work of Broda and Weinstein (2006), who provide tens of thousands of elasticities using 10-digit HS US data, results in a median value of 3.10.

Given our estimates of θ, it is straightforward to back out the Armington elasticity ρ within the context of the model of Anderson and van Wincoop (2004), where $\rho = \theta + 1$. Using our estimates of the elasticity of trade, the implied Armington elasticity ranges between 4.9-5.2. This utility parameter also appears in the heterogeneous firm framework of Melitz (2003) parameterized by Chaney (2008). Together with the elasticity of trade, θ, the utility parameter governs the distribution of firm sales arising from the model, which has Pareto tales with a slope given by $\theta/(\rho - 1)$. Luttmer (2007) provides firm-level evidence that this slope takes on the value of 1.65, which given our estimates of θ, provides the range of $3.38 - 3.56$ for ρ. Hence, the Armington elasticity implied by our estimates ranges between $3.38 - 5.2$, which falls within the low end of the ranges of estimates of existing studies.

8. Robustness

8.1. The Number of Goods

To be completed

8.2. Country-Specific Taxes and Distribution Costs

The price data used in our estimation is collected at the retail level. As such, it necessarily reflects local (distribution) costs and sales taxes. It turns out that these market frictions do not affect our estimates of the elasticity parameter, for as long as they are country- but not good-specific. To see this, suppose consumers in destination n must pay a marginal sales tax $\tau_n - 1$ on each product. Alternatively, $\tau_n - 1$ can also be thought of as a destination-specific marginal retail cost. Under these assumptions, the price (inclusive of taxes) a consumer in destination n pays for product j, $p_n^T(j)$, within the context of the model of Eaton and Kortum (2002) becomes:

$$p_n^T(j) = \tau_n \min_{k=1,\dots,N} \{p_{nk}(j)\}.$$

Substituting the pricing rule into the productivity distribution allows us to obtain the following price index for each destination n:

$$p_n^T = \tau_n \gamma \left[\sum_{k=1}^{N} T_k (\tau_{nk} w_k)^{-\theta} \right]^{-\frac{1}{\theta}}. \tag{18}$$

The expression for a trade share remains unchanged as all products sold in destination n are taxed uniformly:

$$\frac{X_{ni}}{X_n} = \frac{T_i(\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^{N} T_k(\tau_{nk} w_k)^{-\theta}}. \tag{19}$$

Expressions (18) and (19) yield:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{\tau_{ni} p_i^T / \tau_i}{p_n^T / \tau_n} \right)^{-\theta} = \left(\frac{\tau_{ni} p_i}{p_n} \right)^{-\theta}, \tag{20}$$

which is equivalent to expression (3).

So, from the model's perspective, sales taxes should not affect estimates of the key parameter.

In order to estimate the parameter θ, however, we must first arrive at a measure of trade frictions. If the price data we observe include sales taxes, the measured trade friction exporters

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from i face in order to serve destination n also reflects these taxes:

$$\hat{\tau}_{ni} = \frac{\tau_n}{\tau_i} \max_{\ell} \left\{ \frac{p_n(\ell)}{p_i(\ell)} \right\}. \tag{21}$$

Using $\hat{\tau}_{ni}$ in (20) would necessarily change the estimate of θ, should the pre-tax price indices, p_i, be used. However, if we use the observed price indices, which include taxes, together with $\hat{\tau}_{ni}$, expression (20) becomes:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{\hat{\tau}_{ni} p_i^T}{p_n^T} \right)^{-\theta} = \left(\frac{\tau_n \tau_{ni} \tau_i p_i}{\tau_i \tau_n p_n} \right)^{-\theta},$$

which reduces to (3).

Sales taxes that appear in observed price data are completely offset by the estimated trade barriers using these data, thus yielding identical estimates of the elasticity parameter as in the benchmark model. Hence, the presence of local taxes or distribution costs does not bias our estimates of the elasticity of trade.

8.3. Measurement Error

To be completed

8.4. Mark-ups

The price data used in our estimation likely reflects retail mark-ups, which are not only country-, but also retailer-specific. In order to check whether such variable mark-ups affect our results, we make use of a richer price dataset. In particular, we obtain price data provided by the EIU Worldwide Cost of Living Survey, which spans 77 of the original 123 countries we consider. More importantly, the data comprises of 111 tradable goods per country, and the price of each product is recorded once in a supermarket and once in a mid-price store. We repeat our exercise by first using the prices of items collected in mid-price stores, which appear to be cheaper on average, and then the prices found in supermarkets or chain stores. We postpone the results until section 8.5.1 below, since the level of detail in the EIU data allows us to also address aggregation bias, described there.

8.5. Aggregation

8.5.1. Data Approach

The basic-heading data employed in our analysis constitute fairly disaggregate price data, however, the data do not represent individual good price observations. For example, a price observation titled "rice" contains the average price across different types of rice sampled, for example basmati rice, wild rice, whole-grain white rice, etc. Since estimating the elasticity parameter necessitates arriving at a measure of trade barriers via the maximum price difference across observed good prices for each pair of countries, the elasticity estimate may be biased upwards due to a downward bias in trade barrier estimates arising from aggregation. To see this, suppose that for importer n, basmati rice is the binding good that allows us to estimate the trade cost of importing from country i. In the ICP data however, we only observe the average price of rice which reflects prices of multiple varieties of rice. Hence, the difference between the average prices of rice between the two countries is necessarily smaller than the price difference of basmati rice, should the remaining types of rice be more equally priced across the two countries. In this case, trade barriers are underestimated and consequently the elasticity of trade is biased upwards.[13]

Table 8: Estimation Results With EIU Data

Cheap Stores, Overidentified		
Estimation Approach	Estimate of θ	Standard Error
SMM	2.56	???
Method of Moments, β_1	4.17	0.03
Method of Moments, β_2	5.11	0.03
Expensive Stores, First Order Statistic		
SMM	2.63	???
Method of Moments, β_1	4.39	0.03
Method of Moments, β_2	5.23	0.03

In order to alleviate the aggregation problem, we present estimates of the elasticity parameter stemming from the good-level price dataset provided by the Economist Intelligence Unit (EIU), which spans a subset of 77 countries from our original dataset, but provides prices

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[13]Our aggregation argument is different from the argument by Imbs and Mejean (2009) who demonstrate that imposing elasticities across disaggregated sectors of the economy to be equivalent results in lower elasticity of substitution estimates than ones obtained by allowing for heterogeneity.

for 111 individual tradable goods in two types of retail stores.

The results in table 8 suggest that aggregation causes a downward bias on trade barrier estimates, resulting in elasticity of trade estimates that are biased upwards. Indeed, when we use the highly disaggregate EIU dataset, the elasticity of trade falls to $2.56 - 2.63$. However, retail mark-ups do not seem to bias the estimates, since the elasticity of trade is not dramatically different whether high- or low-end store prices are used.

8.5.2. Model Approach

The aggregation problems discussed above are only reflected in the first-stage of our analysis which applies the Eaton and Kortum (2002) methodology to the actual ICP data. Once we invoke our simulation methodology which makes use of a particular model that features price heterogeneity, we employ simulated good-level prices in order to estimate trade barriers and therefore the elasticity of trade. From the point of view of the Eaton and Kortum (2002) model, aggregating the goods in the basic-heading manner employed by the ICP is not possible. Recall that this one-sector model features a continuum of goods and each good is bought from the cheapest source. So, while varieties of this good which are potentially supplied from a number of sources could be aggregated into a basic heading, only the cheapest one of these varieties is actually supplied to the market and its price is recorded in the data. Hence each good in the data represents a particular basic heading and further aggregation that is consistent with the ICP methodology is impossible.

An aggregation argument potentially goes through in the monopolistic competition framework of Melitz (2003) and Chaney (2008). In these models, varieties produces by firms with identical productivity draws from different source countries can be thought of as varieties of a good produced with a particular productivity level. Thus, a basic heading price represents the average price of all varieties produced by firms with a particular productivity draw originating from different source countries. We are exploring the implications of the monopolistic competition micro-structure on the estimates of the elasticity of trade in a companion paper, Simonovska and Waugh (2010).

9. Why Estimates of θ Matter: The Welfare Gains From Trade

The elasticity parameter θ is key in measuring the welfare gains from trade across all models outlined in this paper. Arkolakis, Costinot, and Rodrguez-Clare (2009) argue that the percentage change in real income necessary to compensate a representative consumer for going

to autarky—or the welfare cost of autarky—is uniquely measured by the share of domestic expenditure in a country and the elasticity of trade parameter.

To understand the argument, recall that all models outlined above rely on a CES representative consumer specification. Hence, welfare gains from trade are essentially captured by changes in the CES price index a representative consumer faces. Unfortunately, data necessary to construct pre- and post-trade CES price indices is unavailable, as we emphasize throughout the text. However, the models generate the following relationship between (unobservable) changes in price indices and (observable) changes in domestic expenditure shares as well as the elasticity parameter:

$$\frac{P'_n}{P_n} - 1 = 1 - \left(\frac{X'_{nn}/X'_n}{X_{nn}/X_n}\right)^{\frac{1}{\theta}}, \tag{22}$$

where the left-hand side can be interpreted as the percentage compensation a representative consumer requires to move from a trade to an autarky equilibrium. Notice that trade liberalization episodes, which imply a relative decrease in the domestic expenditure share of a country, necessarily generate welfare gains by lowering the price index in the particular country.

It is fairly easy to demonstrate that (22) implies that θ represents the inverse of the elasticity of welfare with respect to domestic expenditure shares:

$$\log(P_n) = -\frac{1}{\theta} \log\left(\frac{X_{nn}}{X_n}\right) \tag{23}$$

Hence, decreasing the domestic expenditure share by 1% generates $(1/\theta)/100$ percent increase in consumer welfare. Using the estimates for θ arising from the simple procedure and the improved simulated method of moments procedure, roughly 8 and 4, respectively, the welfare gains from trade would be mis-measured by a hundred percent. Namely, an estimate for θ of 8 would generate 0.125% welfare increase for a percent fall in the domestic share, while an estimate of 4 suggests a 0.25% welfare gain from trade, twice as high as the original calculation. These differences illustrate the importance to obtain better estimates of the elasticity of trade.

10. Conclusion

The methodology in our paper has broader implications than merely arriving at a better estimate of the elasticity of trade. Results from Arkolakis, Costinot, and Rodrguez-Clare

(2009) suggest that heterogenous firm and production models provide no value added for aggregate outcomes over models which abstract from heterogeneity. Our methodological approach suggests otherwise. In this paper, we exploited the structure of the Eaton and Kortum (2002) model to provide a better estimate of the elasticity of trade which is the key parameter to measuring the welfare gains from trade. Our approach would not have been possible in models without heterogenous outcomes. Thus while the Eaton and Kortum (2002), Melitz (2003) and Chaney (2008) models may provide no new additional gains from trade, their structure allows us to provide a better elasticity of trade than a simple Armington model would have allowed. The ability to use both measurement and theory in ways that alternative models would not allow is an important component of the value added that new heterogenous firm and production models of international trade provide.

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11. Models of Trade

11.1. Armington Model Without Heterogeneity

In principal there is nothing unique about equation (3) to the model of Eaton and Kortum (2002). The model of Anderson and van Wincoop (2003) generates equation (3) as well. To do so, assume that each country has constant returns technologies with competitive firms producing a good which is defined by its country of origin, i.e., the Armington assumption. These assumptions imply the unit cost (and price) to deliver a country i good to destination n is $p_{ni} = \tau_{ni} T_i^{-\frac{1}{\theta}} w_i$. Similarly to above, w_i is the unit labor cost in country i and $T_i^{-\frac{1}{\theta}}$ is total factor productivity there.

Preferences are equally simple. Each country has symmetric constant elasticity preferences over all the (country-specific) goods with common elasticity of substitution $\rho = \theta + 1 > 1$. The model yields expenditure shares

$$\frac{X_{ni}}{X_n} = \frac{T_i(\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^{N} T_k(\tau_{nk} w_k)^{-\theta}}. \tag{24}$$

Given preferences, destination n faces the following price index of tradable goods:

$$P_n = \left[\sum_{k=1}^{N} T_k \left(\tau_{nk} w_k \right)^{-\theta} \right]^{-\frac{1}{\theta}}. \tag{25}$$

Expressions (24) and (25) allow us to relate observed expenditure shares to bilateral trade frictions and the price indices of each trading partner via the following equation:

$$\frac{X_{ni}/X_n}{X_{ii}/X_i} = \left(\frac{\tau_{ni} P_i}{P_n} \right)^{-\theta}. \tag{26}$$

This is the same expression as in (3) relating the bilateral trade shares to trade costs and the relative aggregate price of tradables.

11.2. Monopolistic Competition Model With Heterogeneity

Monopolistic competition models of trade in the spirit of Melitz (2003), under the parametrization proposed by Chaney (2008), turn out to generate an identical relationship between

prices, trade frictions and trade flows. As in previous sections, consumers are assumed to derive utility from the consumption of varieties originating from different source countries, combined in an aggregate symmetric CES bundle with constant elasticity of substitution $\rho > 1$. Each variety, however, is produced by a single firm, where firms are differentiated by their productivity, z, and country of origin, i. In every country i, there exists a pool of potential entrants who incur a fixed cost, $e_i > 0$, in domestic wages, and subsequently draw a productivity from a Pareto distribution, $T_i z^{-\theta}$, with support $[T_i^{1/\theta}, \infty)$. Only a measure J_i of them enter in equilibrium and firm entry and exit drives average profits in each country to zero. Finally, firms need to incur fixed market access costs (in destination wages) to reach destination n, f_n. Thus only a subset of them, $N_{ni} = J_i T_i / (z_{ni}^*)^\theta$, access each market, where z_{ni}^* denotes the productivity threshold for successful firms from i in n.

θ plays the same role in the model of Melitz (2003) and Chaney (2008) as it does in the model of Eaton and Kortum (2002) because the two frameworks assume identical preference structures and rely of productivity distributions that are tightly linked. To illustrate the latter point, we re-examine an argument made by Eaton, Kortum, and Kramarz (2008). Suppose that agents consume varieties indexed by ω, where each variety is produced with efficiency $z \in [0, J]$. Let the measure of varieties produced with efficiency of at least z be given by:

$$f(z; J) = J \left\{ 1 - \exp\left[-\frac{T}{J} z^{-\theta} \right] \right\} \tag{27}$$

If $J = 1$, (27) collapses to the Fréchet distribution used by Eaton and Kortum (2002). If on the other hand $J \to \infty$, (27) becomes the Pareto distribution with shape parameter θ, used in Chaney (2008) and Helpman, Melitz, and Yeaple (2004). To see this, rewrite (27) and apply the L'Hôpital rule as follows:

$$\lim_{J \to \infty} J \left\{ 1 - \exp\left[-\frac{T}{J} z^{-\theta} \right] \right\}$$

$$= \lim_{J \to \infty} \frac{\left\{ 1 - \exp\left[-\frac{T}{J} z^{-\theta} \right] \right\}}{J^{-1}}$$

$$= \lim_{J \to \infty} \frac{\left\{ \exp\left[-\frac{T}{J} z^{-\theta} \right] \right\} z^{-\theta} \frac{T}{J^2}}{J^{-2}}$$

$$= \lim_{J \to \infty} \left\{ \exp\left[-\frac{T}{J} z^{-\theta} \right] \right\} z^{-\theta} T$$

$$= z^{-\theta} T$$

Thus, θ governs the variability in the distribution of productivities in both Ricardian and monopolistic competition frameworks.

Under the assumption of Pareto-distributed productivities, the model of Melitz (2003) and Chaney (2008) gives rise to the following expenditure share for each destination n on goods from source i:

$$\frac{X_{ni}}{X_n} = \frac{J_i T_i (\tau_{ni} w_i)^{-\theta}}{\sum_{k=1}^{N} J_k T_k (\tau_{nk} w_k)^{-\theta}}, \tag{28}$$

where the equilibrium number of entrants is proportional to the fixed cost of entry in each country, $J_i = (\rho - 1)/\rho\theta L_i/e_i$. Given preferences, destination n faces the following price index of tradable goods:

$$P_n = \Upsilon \left[\sum_{k=1}^{N} J_k T_k (\tau_{nk} w_k)^{-\theta} \right]^{-\frac{1}{\theta}} \left(\frac{f_n}{L_n} \right)^{\frac{-\theta-1+\rho}{-\theta(\rho-1)}}, \tag{29}$$

where Υ contains constant terms. Assuming that market access costs are proportional to market size, $(\forall k) f_k = A L_k$, equations (28) and (29) yield expression (3) as in the model of Eaton and Kortum (2002) and (26) using the Armington model.

11.3. Monopolistic Competition Model Without Heterogeneity

Variants of the monopolistic competition model of Krugman (1980) also generate an identical relationship between prices, trade frictions and trade flows as above. These models can be thought of as assuming degenerate firm productivity distributions in the frameworks of Melitz (2003) and Chaney (2008) outlined above. Moreover, they give rise to trade shares and prices that much resemble the ones suggested by the Armington Ricardian model of Anderson and van Wincoop (2003). Hence, expression (3) or (26) follows.

12. Data Appendix

12.1. Trade Shares

To construct trade shares, we used bilateral trade flows and production data in the following way:

$$\frac{X_{ni}}{X_n} = \frac{\text{Imports}_{ni}}{\text{Gross Mfg. Production}_n - \text{Total Exports}_n + \text{Imports}_n},$$

$$\frac{X_{nn}}{X_n} = 1 - \sum_{k \neq n}^{N} \frac{X_{ni}}{X_n}.$$

Putting the numerator and denominator together is simply computing an expenditure share by dividing the value of goods country n imported from country i by the total value of goods in country n. The home trade share $\frac{X_{nn}}{X_n}$ is simply constructed as the residual from one minus the sum of all bilateral expenditure shares.

To construct $\frac{X_{ni}}{X_n}$, the numerator is the aggregate value of manufactured goods that country n imports from country i. Bilateral trade flow data are from UN Comtrade for the year 2004. We obtain all bilateral trade flows for our sample of 123 countries at the four-digit SITC level. We then used concordance tables between four-digit SITC and three-digit ISIC codes provided by the UN and further modified by Muendler (2009).[14] We restrict our analysis to manufacturing bilateral trade flows only, namely, those that correspond with manufactures as defined in ISIC Rev.#2.

The denominator is gross manufacturing production minus total manufactured exports (for the whole world) plus manufactured imports (for only the sample). Gross manufacturing production data are the most serious data constraints we faced. We obtain manufacturing production data for 2004 from UNIDO for a large sub-sample of countries. We then imputed gross manufacturing production for countries for which data are unavailable as follows: We first obtain 2004 data on manufacturing (MVA) and agriculture (AVA) value added as well as population size (L) and GDP for all countries in the sample. We then impute the gross output (GO) to manufacturing value added ratio for the missing countries using coefficients

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[14]The trade data often report bilateral trade flows from two sources. For example, the exports of country A to country B can appear in the UN Comtrade data as exports reported by country A or as imports reported by country B. In this case, we take the report of bilateral trade flows between countries A and B that yields a higher total volume of trade across the sum of all SITC four-digit categories.

resulting from the following regression:

$$\log\left(\frac{MVA}{GO}\right) = \beta_0 + \beta_{GDP}C_{GDP} + \beta_L C_L + \beta_{MVA}C_{MVA} + \beta_{AVA}C_{AVA} + \epsilon,$$

where β_x is a 1x3 vector of coefficients corresponding to C_x, an Nx3 matrix which contains $[\log(x), (\log(x))^2, (\log(x))^3]$ for the sub-sample of N countries for which gross output data are available.

12.2. Prices

The ICP price data we employ in our estimation procedure is reported at the basic-heading level. Here we discuss briefly how these prices are collected. An issue we discuss is that the prices in the data are aggregates over even more detailed products. In our estimation routine we abstracted from this issue. However, we should emphasize that a key advantage of our simulated method of moments procedure is that these aggregation problems can be explicitly addressed.

The basic heading level represents a narrowly-defined group of goods for which expenditure data are available. For example, basic heading "1101111 Rice" is made up of prices of different types of rice and the resulting value is an aggregate over these different types of rice. This implies that a typical price observation of "Rice" contains different types of rice as well as different packaging options that affect the unit price of rice within and across countries.

According to the ICP Handbook, the price of the basic heading "Rice" is constructed using a transitive Jevons index of prices of different varieties of rice. To illustrate this point, suppose the world economy consists of 3 countries, A, B, C and 10 types of rice, 1-10. Further suppose that consumers in country 1 have access to all 10 types of rice; those in country 2 only have access to types 1-5 of rice; and those in country 3 have access to types 4-6 of rice. Although all types of rice are not found in all 3 countries, it is sufficient that each pair of countries shares at least one type of rice.

The ICP obtains unit prices for all available types of rice in all three countries and records a price of 0 if the type of rice is not available in a particular country. The relative price of rice between countries 1 and 2, based on goods available in these two countries, $p_{AB}^{A,B}$, is a geometric average of the relative prices of rice of types $1 - 5$

$$p_{AB}^{A,B} = \left[\prod_{j=1}^{5} \frac{p_A(j)}{p_B(j)}\right]^{\frac{1}{5}}.$$

Similarly, one can compute the relative price of rice between countries A and C (B and C) based on varieties available in both A and C (B and C). The price of the basic heading "Rice" reported by the ICP is:

$$p_{AB} = \left[p_{AB}^{A,B} p_{AB}^{A,B} \frac{p_{AC}^{A,C}}{p_{BC}^{B,C}} \right]^{\frac{1}{3}},$$

which is a geometric average that features not only relative prices of rice between countries A and B, but also cross-prices between A and B linked via country C. This procedure ensures that prices of basic headings are transitive across countries and minimizes the impact of missing prices across countries.

Thus, a basic heading price is a geometric average of prices of varieties that is directly comparable across countries.

13. Proofs and Tables

13.1. Productivity and Marginal Cost Distribution

Proposition 3 *If $z_i \sim F_i(z_i) = \exp(-T_i z_i^{-\theta})$, then $u_i \equiv z_i/w_i \sim G_i(u_i) = \exp(-\tilde{S}_i u_i^{-\theta})$, where $\tilde{S}_i = T_i w_i^{-\theta}$.*

Proof Let $z_i \sim F_i(z_i) = \exp(-T_i z_i^{-\theta})$ and define $u_i \equiv z_i/w_i$. The pdf of z_i, $f_i(z_i) = \exp(-T_i z_i^{-\theta})\theta T_i z_i^{-\theta-1}$. To find the pdf of the transformation u_i, $g_i(u_i)$, recall that is must be that $f_i(z_i)dz_i = g_i(u_i)du_i$, or $g_i(u_i) = f_i(z_i)(du_i/dz_i)^{-1}$. Let $\tilde{S}_i = T_i w_i^{-\theta}$. Using $f_i(z_i)$, $\tilde{S}_i$, and the fact that $du_i/dz_i = 1/w_i$, we obtain:

$$g_i(u_i) = f_i(z_i) \left(\frac{du_i}{dz_i} \right)^{-1}$$

$$= \exp(-T_i z_i^{-\theta})\theta T_i z_i^{-\theta-1} \left(\frac{1}{w_i} \right)^{-1}$$

$$= \exp \left(-T_i z_i^{-\theta} \frac{w_i^{-\theta}}{w_i^{-\theta}} \right) \theta T_i z_i^{-\theta-1} \left(\frac{1}{w_i} \right)^{-1} \frac{w_i^{-\theta}}{w_i^{-\theta}}$$

$$= \exp \left(-\tilde{S}_i \frac{z_i^{-\theta}}{w_i^{-\theta}} \right) \theta \tilde{S}_i \frac{z_i^{-\theta-1}}{w_i^{-\theta-1}}$$

$$= \exp \left(-\tilde{S}_i u_i^{-\theta} \right) \theta \tilde{S}_i u_i^{-\theta-1}$$

Clearly $g_i(u_i)$ is the pdf that corresponds to the cdf $G_i(u_i) = \exp(-\tilde{S}_i u_i^{-\theta})$, which concludes

the argument.

13.2. Proofs of Propositions 1 and 2

Lemma 1 *If $z \sim \exp(T_i)$, then $y \equiv z^{-\frac{1}{\theta}} \sim \exp(-T_i y^{-\theta})$.*

Proof Since $y = h(z)$ is a decreasing function, it must be that $f(z)dz = -g(y)dy$, where f, g are the pdf's of z, y respectively. The result follows from simple algebra.

The proof of proposition 1 follows.

Proof Let the price for a good z produced in country k and supplied to country i be $p_{ik} \equiv w_k \tau_{ik} z_k^{\frac{1}{\theta}}$. We are interested in the following object:

$$v_{ij}(z) = \frac{\min\left\{\min_{k \neq j}[w_k \tau_{ik} z_k^{\frac{1}{\theta}}], w_j \tau_{ij} z_j^{\frac{1}{\theta}}\right\}}{\min\left\{\min_{k \neq i}[w_k \tau_{jk} z_k^{\frac{1}{\theta}}], w_i \tau_{ji} z_i^{\frac{1}{\theta}}\right\}} \tag{30}$$

Take this object to the power of θ:

$$(v_{ij}(z))^\theta = \frac{\min\left\{\min_{k \neq j}[w_k^\theta \tau_{ik}^\theta z_k], w_j^\theta \tau_{ij}^\theta z_j\right\}}{\min\left\{\min_{k \neq i}[w_k^\theta \tau_{jk}^\theta z_k], w_i^\theta \tau_{ji}^\theta z_i\right\}} \tag{31}$$

We will use properties of the exponential distribution to characterize the distribution of this object, which contains minima of exponentially-distributed variables. First, define $\tilde{z}_{ik} = w_k^\theta \tau_{ik}^\theta z_k$. What is the distribution of this variable, provided that $z_k \sim \exp(T_k)$? The distribution is again exponential but with different location parameter. In particular, using usual rule for increasing transformation, $f(z)dz = g(y)dy$, $\tilde{z}_{ik} \sim \exp(T_k w_k^{-\theta} \tau_{ik}^{-\theta})$. Let $\tilde{\lambda}_{ik} \equiv T_k w_k^{-\theta} \tau_{ik}^{-\theta}$.

Next, we want the distribution of $\min_{k \neq j}[w_k^\theta \tau_{ik}^\theta z_k]$ or equivalently $\min_{k \neq j}[\tilde{z}_{ik}]$. By assumption each z is independently distributed across countries k. Also, we just showed that $\tilde{z}_{ik}$ is exponentially distributed. We will now use a rule for the distribution of the minimum of a sequence of independently exponentially distributed r.v.'s.

Define $\tilde{z}_i \equiv \min_{k \neq j}[\tilde{z}_{ik}]$. Since each $\tilde{z}_{ik} \sim \exp(\tilde{\lambda}_{ik})$ and independent, $\tilde{z}_i \sim \exp(\sum_{k \neq j} \tilde{\lambda}_{ik})$. Define $\tilde{\lambda}_i \equiv \sum_{k \neq j} \tilde{\lambda}_{ik}$. Do the same thing for j in the denominator.

Rewrite expression as:

$$(v_{ij}(z))^\theta = \frac{\min\left\{\tilde{z}_i, w_j^\theta \tau_{ij}^\theta z_j\right\}}{\min\left\{\tilde{z}_j, w_i^\theta \tau_{ji}^\theta z_i\right\}} \tag{32}$$

Take logs and define $\epsilon_{ij}(z) = \log(v_{ij}(z))$. Expression becomes:

$$\theta\epsilon_{ij}(z) = \min\left\{\log(\tilde{z}_i), [\theta\log(w_j) + \theta\log(\tau_{ij}) + \log(z_j)])\right\} - \min\left\{\log(\tilde{z}_j), [\theta\log(w_i) + \theta\log(\tau_{ji}) + \log(z_i)])\right\} \tag{33}$$

Now, we want to find the truncation points (min,max) of the distribution of object $\epsilon_{ij}(z)$. Finally, we will also derive its actual distribution.

First, the truncation points. Look at the above object. It describes relative price of the same good z in countries i and j. There are three possibilities:

1. Countries i and j buy good z from two different sources. Then,

$$\theta\epsilon_{ij}(z) = \log(\tilde{z}_i) - \log(\tilde{z}_j) \tag{34}$$

2. Country i buys good z from country j. Assuming trade barriers don't violate triangle inequality (for triangle inequality to hold it must be the case that $\tau_{ij} < \tau_{ik}\tau_{kj}$ for all triplets), then it must be that j buys the good from itself. Then,

$$\theta\epsilon_{ij}(z) = \theta\log(w_j) + \theta\log(\tau_{ij}) + \log(z_j) - \theta\log(w_j) - \log(z_j) = \theta\log(\tau_{ij}) \tag{35}$$

3. Country j buys good z from country i. Then it must be that i buys the good from itself. Then,

$$\theta\epsilon_{ij}(z) = \theta\log(w_i) + \log(z_i) - \theta\log(w_i) - \theta\log(\tau_{ji}) - \log(z_i) = -\theta\log(\tau_{ji}) \tag{36}$$

From these three cases, we need to figure out what the bounds are. Case 1 is a realization of a random variable. Cases 2 and 3 are two actual numbers coming from gravity. We now show that the following ordering occurs: $-\theta\log(\tau_{ji}) < \log(\tilde{z}_i) - \log(\tilde{z}_j) < \theta\log(\tau_{ij})$. To check these inequalities, we need to look at two scenarios:

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1. Countries i and j buy good z from the same source k. Then,

$$\log(\tilde{z}_i) - \log(\tilde{z}_j) = \log(w_k^\theta \tau_{ik}^\theta z_k) - \log(w_k^\theta \tau_{jk}^\theta z_k)$$
$$= \theta(\log(\tau_{ik}) - \log(\tau_{jk})) \tag{37}$$

Clearly,

$$\theta(\log(\tau_{ik}) - \log(\tau_{jk})) \geq -\theta \log(\tau_{ji})$$
$$\iff \theta \log(\tau_{ik}) + \theta \log(\tau_{ji}) \geq \theta \log(\tau_{jk})$$
$$\iff \tau_{ji}\tau_{ik} \geq \tau_{jk} \tag{38}$$

where the latter inequality is true under the triangle inequality assumption.

Similarly,

$$\theta(\log(\tau_{ik}) - \log(\tau_{jk})) \leq \theta \log(\tau_{ij})$$
$$\iff \theta \log(\tau_{ik}) \leq \theta \log(\tau_{jk}) + \theta \log(\tau_{ij})$$
$$\iff \tau_{ik} \leq \tau_{ij}\tau_{jk} \tag{39}$$

again true by triangle inequality.

2. Country i buys good z from source a and country j from source b, $a \neq b$. We want to show that $-\theta \log(\tau_{ji}) \leq \log(w_a^\theta \tau_{ia}^\theta z_a) - \log(w_b^\theta \tau_{jb}^\theta z_b) \leq \theta \log(\tau_{ij})$.

Since i imported from a over b, it must be that:

$$w_a^\theta \tau_{ia}^\theta z_a \leq w_b^\theta \tau_{ib}^\theta z_b \tag{40}$$

Similarly, since j imported from b over a, it must be that:

$$w_b^\theta \tau_{jb}^\theta z_b \leq w_a^\theta \tau_{ja}^\theta z_a \tag{41}$$

First, I will show the upper bound. Take logs of (40) and subtract $\log(w_b^\theta \tau_{jb}^\theta z_b)$ from both sides:

$$\log(w_a^\theta \tau_{ia}^\theta z_a) - \log(w_b^\theta \tau_{jb}^\theta z_b) \leq \log(w_b^\theta \tau_{ib}^\theta z_b) - \log(w_b^\theta \tau_{jb}^\theta z_b) \tag{42}$$

It suffices to show that the right hand side is itself below the upper bound, since by

transitivity so is the left hand side (which is the object of interest).

$$\log(w_b^\theta \tau_{ib}^\theta z_b) - \log(w_b^\theta \tau_{jb}^\theta z_b) \leq \theta \log(\tau_{ij})$$
$$\Longleftrightarrow \theta \log(\tau_{ib}) - \theta \log(\tau_{jb}) \leq \theta \log(\tau_{ij})$$
$$\Longleftrightarrow \tau_{ib} \leq \tau_{ij}\tau_{jb} \tag{43}$$

which is true by triangle inequality.

The argument for the lower bound is similar. Take logs of (41), multiply by -1 (and reverse inequality) and add $\log(w_a^\theta \tau_{ia}^\theta z_a)$ to both sides:

$$\log(w_a^\theta \tau_{ia}^\theta z_a) - \log(w_b^\theta \tau_{jb}^\theta z_b) \geq \log(w_a^\theta \tau_{ia}^\theta z_a) - \log(w_a^\theta \tau_{ja}^\theta z_a) \tag{44}$$

It suffices to show that the right hand side is itself above the lower bound, since by transitivity so is the left hand side (which is the object of interest).

$$\log(w_a^\theta \tau_{ia}^\theta z_a) - \log(w_a^\theta \tau_{ja}^\theta z_a) \geq -\theta \log(\tau_{ji})$$
$$\Longleftrightarrow \theta \log(\tau_{ia}) - \theta \log(\tau_{ja}) \geq -\theta \log(\tau_{ji})$$
$$\Longleftrightarrow \tau_{ji}\tau_{ia} \geq \tau_{ja} \tag{45}$$

which is true by triangle inequality.

Hence, $\theta \epsilon_{ij}(z) \in [-\theta \log(\tau_{ji}), \theta \log(\tau_{ij})]$, which are truncation points coming straight out of data via gravity.

Next we want to derive the distribution of $\theta \epsilon_{ij}(z) = \log(\tilde{z}_i) - \log(\tilde{z}_j)$. First we derive the pdf's of its two components.

Let $y_i \equiv \log(\tilde{z}_i)$. Then $\tilde{z}_i = \exp(y_i)$. The pdf of y_i must satisfy:

$$f(y_i)dy_i = g(\tilde{z}_i)d\tilde{z}_i$$
$$\Rightarrow f(y_i) = \tilde{\lambda}_i \exp(-\tilde{\lambda}_i \tilde{z}_i)\frac{d\tilde{z}_i}{dy_i}$$
$$\Rightarrow f(y_i) = \tilde{\lambda}_i \exp(-\tilde{\lambda}_i \tilde{z}_i)\tilde{z}_i$$
$$\Rightarrow f(y_i) = \tilde{\lambda}_i \exp(-\tilde{\lambda}_i \exp(y_i)) \exp(y_i)$$
$$\Rightarrow F(y_i) = 1 - \exp(-\tilde{\lambda}_i \exp(y_i)) \tag{46}$$

Similarly for j.

Now that we have the pdf's of the two components, we can define the pdf of $\epsilon \equiv \theta \epsilon_{ij}(z) \in$

$[-\theta \log(\tau_{ji}), \theta \log(\tau_{ij})]$ as follows:

$$f(\epsilon) \equiv f_{y_i - y_j}(x) = \int_{-\infty}^{\infty} f_{y_i}(y) f_{y_j}(y - x) dy \tag{47}$$

where we have used the fact that y_i and y_j are independently distributed, hence the pdf of their difference is the convolution of the pdf's of the two r.v.'s.

Substituting the pdf's of y_i and y_j into (47) yields:

$$f(\epsilon) = \int_{-\infty}^{\infty} \tilde{\lambda}_i \exp(-\tilde{\lambda}_i \exp(y)) \exp(y) \tilde{\lambda}_j \exp(-\tilde{\lambda}_j \exp(y - \epsilon)) \exp(y - \epsilon) dy$$

$$= \frac{-\tilde{\lambda}_i \tilde{\lambda}_j}{(\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j)^2} \left[\frac{\tilde{\lambda}_i \exp(y + \epsilon) + \tilde{\lambda}_j \exp(y) + \exp(\epsilon)}{\exp\left\{ \exp(y)(\tilde{\lambda}_i + \tilde{\lambda}_j \exp(-\epsilon)) \right\}} \right]_{y=-\infty}^{y=+\infty} \tag{48}$$

Let $v(y)$ be the expression in the bracket.

$$\lim_{y \to -\infty} v(y) = \frac{0 + 0 + \exp(\epsilon)}{\exp\{0\}} = \exp(\epsilon) \tag{49}$$

For the upper bound, we use the l'Hopital rule:

$$\lim_{y \to \infty} v(y) = \lim_{y \to \infty} \frac{\tilde{\lambda}_i \exp(y + \epsilon) + \tilde{\lambda}_j \exp(y)}{\exp\left\{ \exp(y)(\tilde{\lambda}_i + \tilde{\lambda}_j \exp(-\epsilon)) \right\} \exp(y)(\tilde{\lambda}_i + \tilde{\lambda}_j \exp(-\epsilon))}$$

$$= \lim_{y \to \infty} \frac{\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j}{\exp\left\{ \exp(y)(\tilde{\lambda}_i + \tilde{\lambda}_j \exp(-\epsilon)) \right\} (\tilde{\lambda}_i + \tilde{\lambda}_j \exp(-\epsilon))}$$

$$= 0 \tag{50}$$

Thus (48) becomes:

$$f(\epsilon) = \frac{\tilde{\lambda}_i \tilde{\lambda}_j \exp(\epsilon)}{(\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j)^2} \tag{51}$$

The corresponding cdf is:

$$F(\epsilon) = 1 - \frac{\tilde{\lambda}_j}{\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j} \tag{52}$$

Given the bounds on ϵ, the truncated pdf is:

$$f_T(\epsilon) = \frac{f(\epsilon)}{F(\theta \log(\tau_{ij})) - F(-\theta \log(\tau_{ji}))}$$

$$= \gamma^{-1} \frac{\tilde{\lambda}_i \tilde{\lambda}_j \exp(\epsilon)}{(\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j)^2}, \qquad (53)$$

where:

$$\gamma = -\frac{\tilde{\lambda}_j}{\tilde{\lambda}_i \exp(\theta \log(\tau_{ij})) + \tilde{\lambda}_j} + \frac{\tilde{\lambda}_j}{\tilde{\lambda}_i \exp(-\theta \log(\tau_{ji})) + \tilde{\lambda}_j} \qquad (54)$$

Then, the truncated cdf is:

$$F_T(\epsilon) = \gamma^{-1} \int_{-\theta \log(\tau_{ji})}^{\epsilon} f(t) dt \qquad (55)$$

Now that we have these distributions, we compute order statistics from these distributions and finally characterize the trade barriers we estimate from price data. We will make use of the following formula:

Given N observations drawn from pdf $h(x)$, the pdf of the r-th order statistic (where $r = N$ is the max and $r = 1$ is the min) is:

$$h_r(x) = \frac{N!}{(r-1)!(N-r)!} H(x)^{r-1} (1 - H(x))^{N-r} h(x) \qquad (56)$$

The max reduces to:

$$h_{\max}(x) = N H(x)^{N-1} h(x)$$

With this pdf defined, we can compute the expectation of the maximum statistic:

$$E[\max_N (x_N)] = \int_{-\infty}^{\infty} x h_{\max}(x) dx \qquad (57)$$

Now, we can finally characterize the pdf of the maximum in our problem, its expectation, and the bias.

So far, we have derived the truncated pdf and cdf of $\epsilon = \theta \log(v_{ij}(z))$. Our object of interest is actually $\log(v_{ij}(z)) = \frac{1}{\theta}\epsilon$. So the price difference is proportional to ϵ with proportionality constant $1/\theta$. Similarly, the expected price difference is proportional to the expectation of ϵ, with the same proportionality constant. Finally, the expectation of the maximum log price

difference for N draws is given by:

$$E[\max_{n \in N}(\log(p_i(n)) - \log(p_j(n)))] = \frac{1}{\theta} \int_{-\infty}^{\infty} \epsilon f_{\max}(\epsilon) d\epsilon \tag{58}$$

Since $\epsilon \in [-\theta \log(\tau_{ji}), \theta \log(\tau_{ij})]$, we only need to care about the truncated pdf and cdf of ϵ. Substituting these into the formula for the N-th order statistic pdf gives:

$$\begin{aligned} f_{\max}(\epsilon) &= N F_T(\epsilon)^{N-1} f_T(\epsilon) \\ &= N \left[\gamma^{-1} \int_{-\theta \log(\tau_{ji})}^{\epsilon} f(t) dt \right]^{N-1} \gamma^{-1} \frac{\tilde{\lambda}_i \tilde{\lambda}_j \exp(\epsilon)}{(\tilde{\lambda}_i \exp(\epsilon) + \tilde{\lambda}_j)^2} \end{aligned} \tag{59}$$

Hence, the expected N-th moment of the log price difference is proportional to $1/\theta$, where the proportionality object comes from gravity.

$$E[\max_{n \in N}(\log(p_i(n)) - \log(p_j(n)))] = \frac{1}{\theta} \Psi_{ij}(S, \theta \log(\tau_{ij}), \theta \log(\tau_{ji})), \tag{60}$$

where

$$\Psi_{ij}(N; S, \theta \log(\tau_{ij}), \theta \log(\tau_{ji})) \equiv \int_{-\infty}^{\infty} \epsilon f_{\max}(\epsilon) d\epsilon \tag{61}$$

and $S = (S_1, ..., S_I)$ is a vector of S_i's for each country in the sample $i = 1, ..., I$ and is defined by $S_i = \log(T_i w_i^{-\theta})$.

Now that we have an estimating equation for τ's, we go to the estimating equation for θ.

Recall that Eaton and Kortum (2002) derived the following equation to characterize θ:

$$\log \left(\frac{X_{ij}/X_i}{X_{jj}/X_j} \right) = -\theta \log(P_j) + \theta \log(P_i) - \theta \log(\tau_{ij}) \tag{62}$$

Consider the following measured counterpart to this expression:

$$\log \left(\frac{X_{ij}/X_i}{X_{jj}/X_j} \right) = \beta \left[\frac{1}{N} \sum_{n \in N} [\log(p_i(n)) - \log(p_j(n))] \right] - \beta \left[\max_{n \in N}(\log(p_i(n)) - \log(p_j(n))) \right] \tag{63}$$

We know what the second part looks like because we just derived it above. Similarly, the

first part is just the expected log price difference:

$$E[(\log(p_i(n)) - \log(p_j(n)))] = \frac{1}{\theta} \int_{-\infty}^{\infty} \epsilon f_T(\epsilon) d\epsilon \tag{64}$$

which is once again proportional to $1/\theta$ with proportionality constant:

$$\Omega_{ij}(N; S, \theta \log(\tau_{ij}), \theta \log(\tau_{ji})) \equiv \int_{-\infty}^{\infty} \epsilon f_T(\epsilon) d\epsilon \tag{65}$$

Thus, we can rewrite the estimating equation for β as:

$$\log\left(\frac{X_{ij}/X_j}{X_{jj}/X_j}\right) = \beta \frac{1}{\theta} \Omega_{ij}(N; S, \theta \log(\tau_{ij}), \theta \log(\tau_{ji})) - \beta \frac{1}{\theta} \Psi_{ij}(N; S, \theta \log(\tau_{ij}), \theta \log(\tau_{ji})) \tag{66}$$

Taking an average over all country pairs I in the sample and inverting gives:

$$\beta = -\theta \frac{\sum_i \sum_i \log\left(\frac{X_{ij}/X_i}{X_{jj}/X_j}\right)}{\sum_i \sum_j (\Psi_{ij} - \Omega_{ij})} \tag{67}$$

The proof of proposition 2 follows.

Proof Convergence would happen only if the denominator and the numerator in expression (67) both converge to the same probability limit. Let's start with the numerator. This is just data, it is not a random variable. So the expectation of the numerator is the numerator itself. Then, it remains to argue that the denominator converges to the numerator multiplied by -1. We argue that the probability limit of $\Psi_{ij} - \Omega_{ij}$ is just the expectation of relative trade flows.

$$plim_{N\to\infty} \Psi_{ij}(N) - \Omega_{ij}(N)$$

$$= plim_{N\to\infty} \theta \left[\frac{1}{N} \sum_{n\in N} [\log(p_i(n)) - \log(p_j(n))] \right] - \left[\frac{1}{N} \sum_{n\in N} \max_{n\in N}(\log(p_i(n)) - \log(p_j(n))) \right]$$

$$= \theta E \left[\log(p_i(z)) - \log(p_j(z)) \right] - E \left[\theta \log(\tau_{ij}) \right]$$

$$= - E \left[\log\left(\frac{X_{ij}/X_i}{X_{jj}/X_j}\right) \right] \tag{68}$$

Going from the first to the second line is by definition, for a given discrete number N of observations. Line two to three: the first part comes from the fact that the average is a consistent estimator of the mean. The second part comes from the fact that as $N \to \infty$, the maximum price difference, which is by definition the maximum of ϵ, converges to its upper bound $\theta \log(\tau_{ij})$. To go from line three to four, use (62) in expectation. So $\beta \to \theta$ as $N \to \infty$.